Gujarat University
M.Sc. (Physics) Sem.-II
Phy-410: (Analog Electronics)

Dr. Aditya M. Vora

Associate Professor,

Department of Physics, University School of Sciences,

Gujarat University, Navrangpura, Ahmedabad 380009

(M) 9426956016

E-mail : amvora@gujaratuniversity.ac.in

Unit-I : Power Supply and Regulators

The Operational Amplifier and Regulators

The Operational Amplifier & Applications : active filters, low pass, high pass, band pass, band reject, notch, first and second order filters, comparators, sample and hold circuits, Logarithmic & antilogarithmic amplifiers.

Reference Books

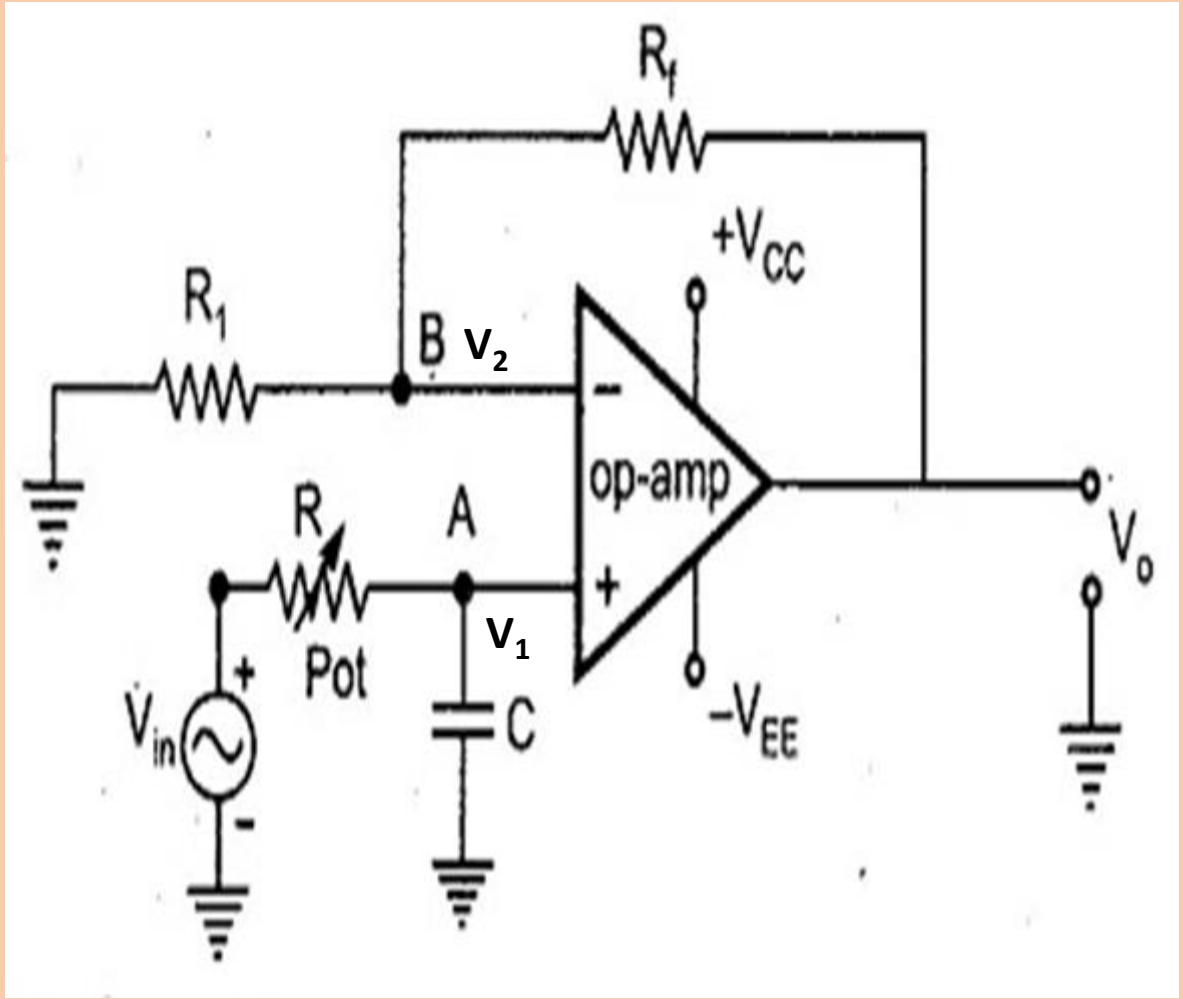
- Integrated circuits, K. R. Botkar, Khanna Publishers.
- Electronic Devices and Circuit Theory, Robert Boylested and Louis Nashelsky, PHI. (Third edition).
- Electronic Fundamentals and Applications: Integrated and Discrete Systems, John D. Ryder, PHI, Fifth edition.
- OPAMP & Linear Integrated Circuits, Ramakant Gayakwad, PHI.

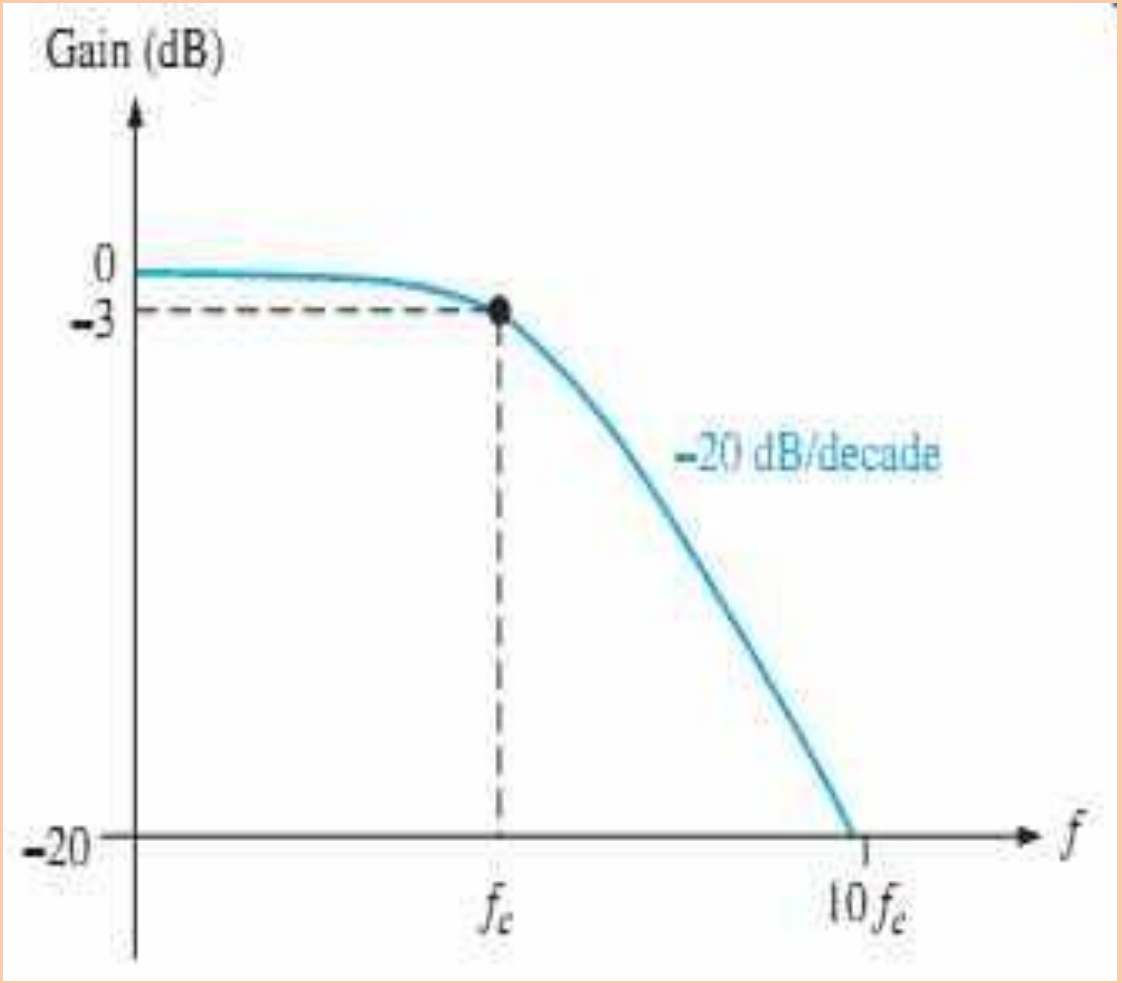
First-Order Low Pass Butterworth Filter

- The Butterworth filter is a type of signal processing filter designed to have a frequency response as flat as possible in the passband. It is also referred to as a maximally flat magnitude filter. It was first described in 1930 by the British engineer and physicist Stephen Butterworth in his paper entitled "On the Theory of Filter Amplifiers".
- Butterworth had a reputation for solving "impossible" mathematical problems. At the time, filter design required a considerable amount of designer experience due to limitations of the theory then in use. The filter was not in common use for over 30 years after its publication. Butterworth stated that:

"*An ideal electrical filter should not only completely reject the unwanted frequencies but should also have uniform sensitivity for the wanted frequencies".*

- Such an ideal filter cannot be achieved, but Butterworth showed that successively closer approximations were obtained with increasing numbers of filter elements of the right values. At the time, filters generated substantial ripple in the passband, and the choice of component values was highly interactive.


R_f
$+V_{CC}$
R_1
B
V_2
op-amp
R
A
V_o
V_1
Pot
V_{in}
C
$-V_{EE}$


Gain (dB)
0
-3
-20 dB/decade
-20
f
f_c
$10f_c$

- In first order low-pass Butterworth filter has only one RC network. It is connected in noninverting configuration of OPAMP because of not load down the RC network. Here, R_1 and R_f determine the gain of the filter.
- Applying the voltage divider rule to the circuit,

$$v_1 = \frac{-jX_c}{R-jX_c} v_{in}. \qquad (1)$$

Where, $j = \sqrt{1}$ and $-jX_c = \frac{1}{2\pi fC}$.

After simplifying the equation (1), we get $v_1 = \frac{v_{in}}{1+j2\pi fRC}$. (2)

And the output voltage is

$$v_{out} = \left(1 + \frac{R_f}{R_1}\right) v_1. \qquad (3)$$

Put the values of v_1 from (2) one can get

$$v_{out} = \left(1 + \frac{R_f}{R_1}\right) \frac{v_{in}}{1+j2\pi f RC}. \qquad (4)$$

Or $$\frac{v_{out}}{v_{in}} = \frac{A_F}{1+j\left(\frac{f}{f_C}\right)}. \qquad (5)$$

Here from equation (5)

i.e. $\frac{v_{out}}{v_{in}} = \frac{A_F}{1+j\left(\frac{f}{f_C}\right)}$,

one can write followings

- $\frac{v_{out}}{v_{in}}$ = gain of the filter as a function of frequency,
- $A_F = \left(1 + \frac{R_f}{R_1}\right)$=passband gain of the filter,
- f = frequency of the input signal, and
- $f_C = \frac{1}{2\pi RC}$ = high cutoff frequency of the filter.

- However, the gain magnitude and phase angle of this filter is obtained by following equations converting them into its equivalent polar form.

Gain magnitude $\rightarrow \left|\frac{v_{out}}{v_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_C}\right)^2}}$. (6)

Phase angle $\rightarrow \phi = tan^{-1}\left(\frac{f}{f_C}\right)$. (7)

The phase angle is measured in degree.

- The operation of such filter can be verified from the gain magnitude equations as follows.

1. At very low frequencies i.e. $f < f_C \rightarrow \left|\frac{v_{out}}{v_{in}}\right| \cong A_F$.
2. At $f = f_C \rightarrow \left|\frac{v_{out}}{v_{in}}\right| \cong \frac{A_F}{\sqrt{2}} = 0.707A_F$.
3. At $f > f_C \rightarrow \left|\frac{v_{out}}{v_{in}}\right| < A_F$.

Hence, such filter has a constant gain A_F from 0Hz to the high cutoff frequency f_C. At that frequency, the gain is $0.707A_F$ and after that it decrease at a constant rate with an increase in frequency.

When the frequency is increased tenfold (one decade), the voltage gain is divided by 10. Therefore, the gain decreases 20dB (=20log10) each time, the frequency is increased by 10. Hence, the rate at which the gain rolls off after f_C is 20dB/decade or 6dB/octave, here octave signifies the twofold increase in frequency. The frequency $f = f_C$ is called cutoff frequency because the gain of the filter at this frequency is down by 3dB (=20log0.707) from 0Hz.

Filter Design

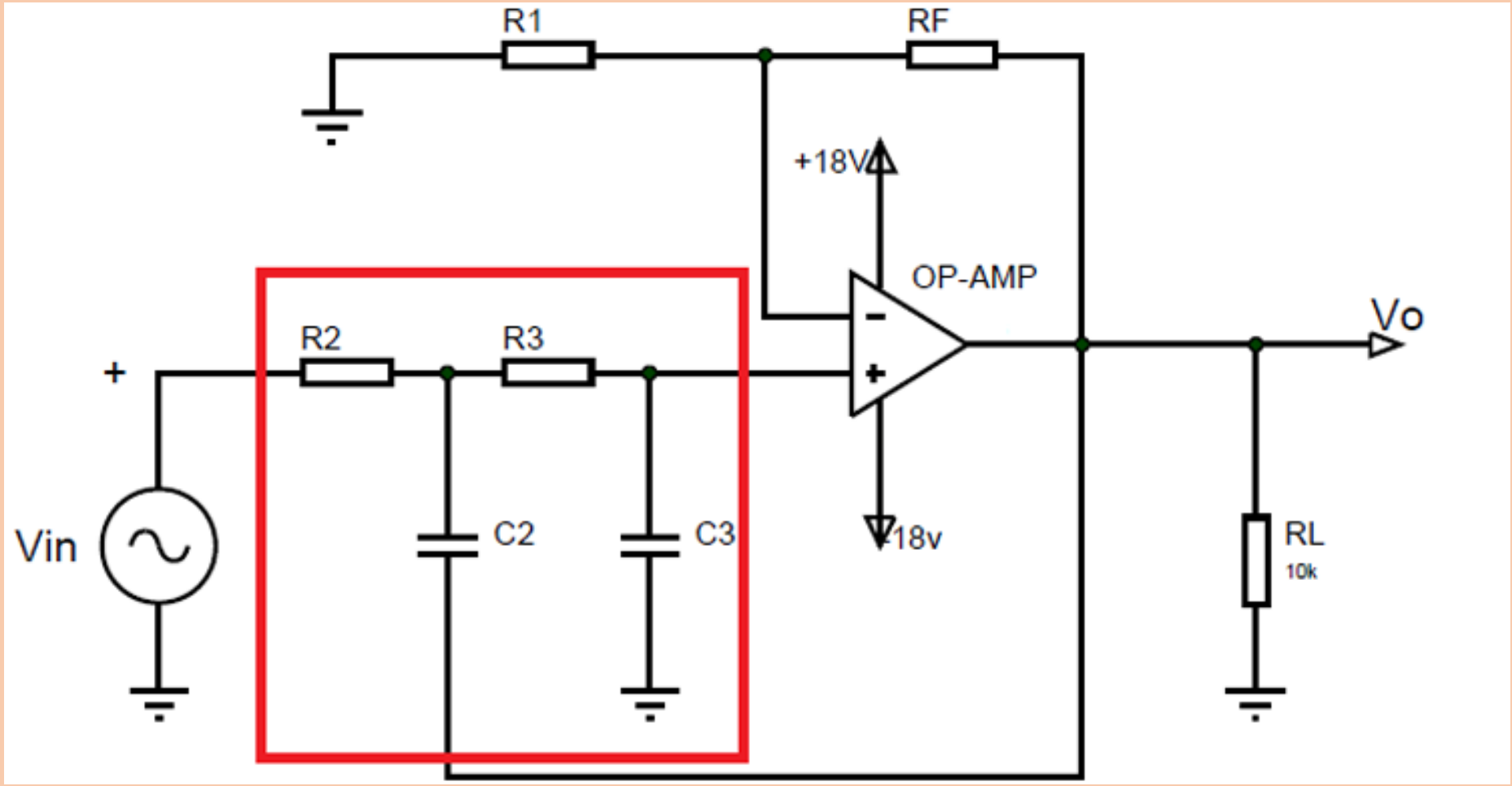
- The low-pass filter can be designed by implementing the following ways :
  1. Choose a value of high cutoff frequency f_C.
  2. Select a value of $C \leq 1\mu F$. Mylar or tantalum capacitors are preferable for better performance.
  3. Calculate the values of R from $R = \frac{1}{2\pi f_C C}$.
  4. Finally, select the values of R_1 and R_F dependent on the desired passband gain A_F by using $A_F = \left(1 + \frac{R_f}{R_1}\right)$.

Frequency Scaling

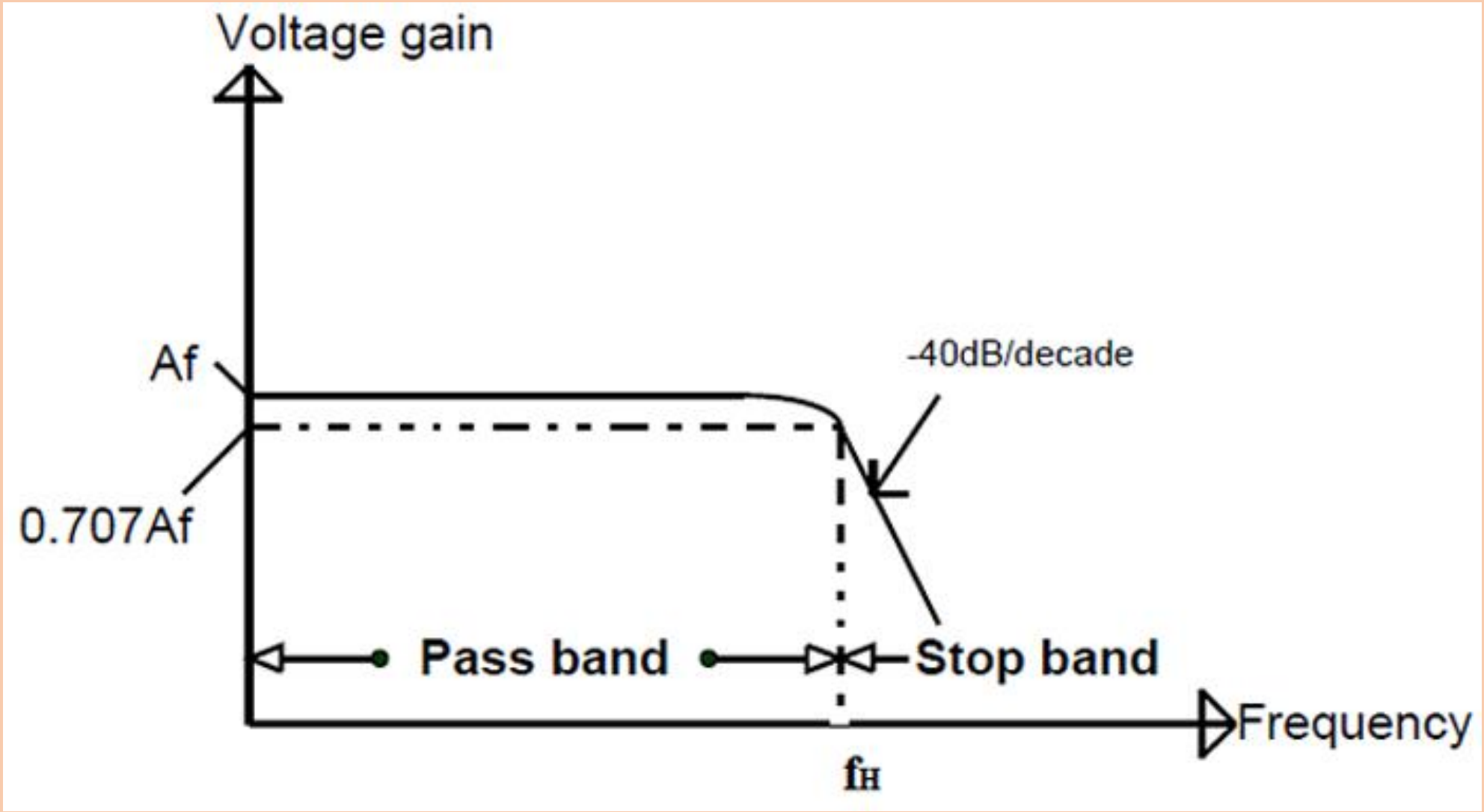
- The frequency scaling is needed sometimes to change the cutoff frequency when filter is designed. The procedure for that, to convert original cutoff frequency f_C to a new cutoff frequency f_C' known as scaling frequency. To do this procedure, multiply R or C, but not both by the ratio of the original cutoff frequency to the new cutoff frequency. Particularly, for filter designing, the needed values of R and C are often not standard. Also, the variable capacitor C is not used commonly. Hence, choose a standard values of C and then calculate the values of R for a desired cutoff frequency. It is because for a nonstandard value of a resistor a potentiometer can be used.

Second Order Low-Pass Butterworth Filter

- An additional RC network connected to the first order Butterworth low-pass filter gives us a second order low-pass filter. This second order low-pass filter has an advantage that the gain rolls-off very fast after the cutoff frequency, in the stop band. A stopband response having a 40 dB/decade roll off is obtained with this filter.
- Design of such filter is more important because higher order filters can be designed by using them.



https://circuitdigest.com/



https://circuitdigest.com/

Here the gain of such filter is set by R_1 and R_f, while the high cutoff frequency f_H is determined by R_2, C_2, R_3 and C_3 as follows

$$f_H = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}. \qquad (8)$$

The voltage gain magnitude of such filter is written as

$$\text{Gain magnitude} \rightarrow \left|\frac{v_{out}}{v_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_H}\right)^4}}. \qquad (9)$$

Here, $A_F = \left(1 + \frac{R_f}{R_1}\right)$=passband gain of the filter,

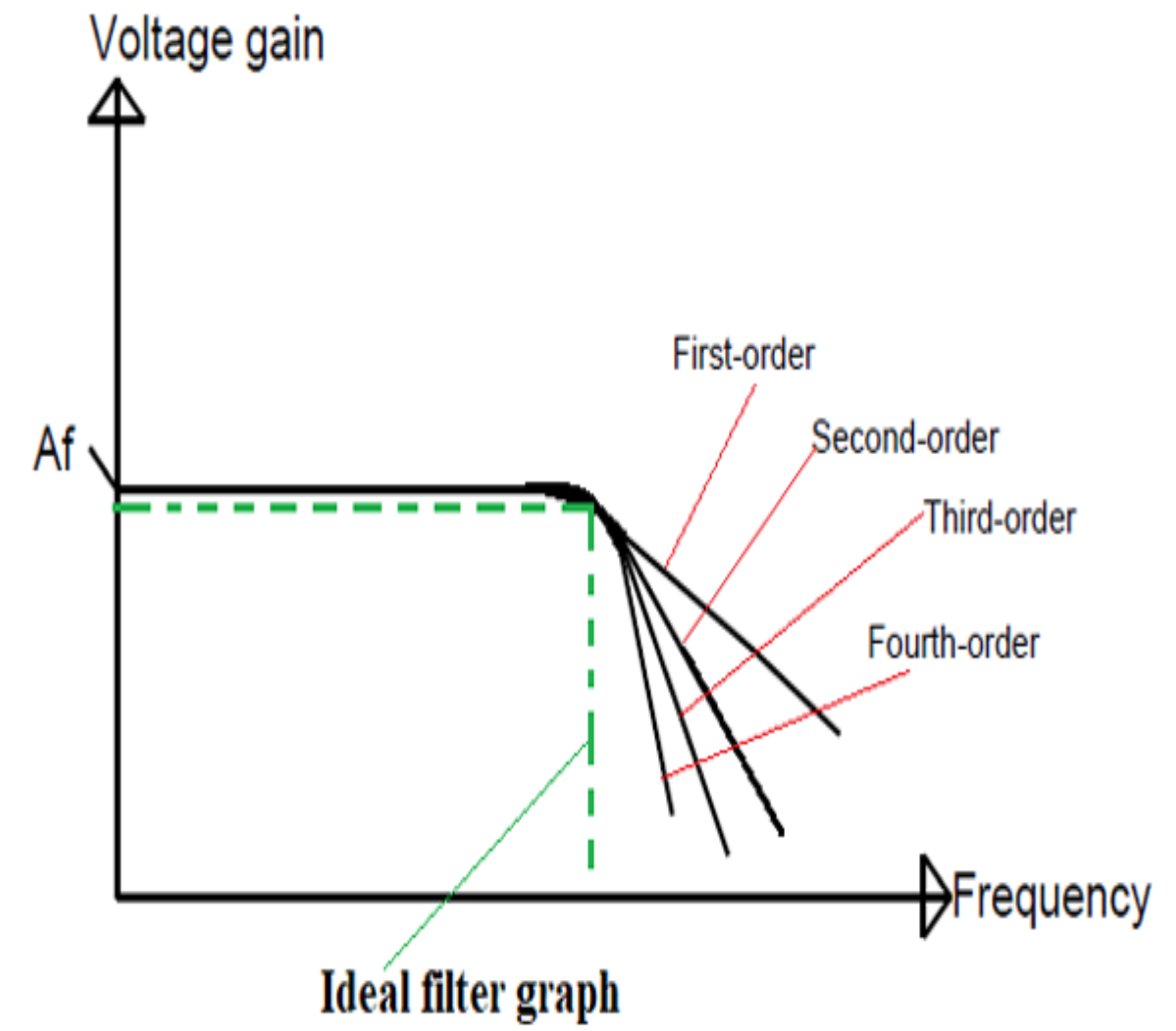
- f = frequency of the input signal (Hz), and
- $f_H = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$ = high cutoff frequency of the filter (Hz).

Filter Design

- Here, except for having twice the roll off rate in the stopband, the frequency response of such filter is identical to that of the first order one. Following steps are identical to those of the first order filter as follows
  1. Choose a value of high cutoff frequency f_H.
  2. To simplify the designed calculations, set R_2=R_3=R and C_2=C_3=C.
  3. Select a value of $C \leq 1\mu F$. Mylar or tantalum capacitors are preferable for better performance.
  4. Calculate the values of R from $R = \frac{1}{2\pi f_H C}$.
  5. Finally, because of the equal resistor and capacitor values, the passband voltage gain $A_F = \left(1 + \frac{R_f}{R_1}\right)$ of the second order low-pass filter has to be equal to 1.586 i.e. R_f=0.586R_1. This gain is necessary to guarantee Butterworth response. Therefore, choose the values of $R_1 \leq 100k\Omega$ and calculate the value of R_f.

Frequency Scaling

- The frequency scaling method of the first order low-pass filter is also applicable to the second order low-pass filter.



https://circuitdigest.com/

In the graph, the green curve represents the ideal filter curve and you can see as the order of the Butterworth filter increases its gain graph leans more towards the ideal curve. So higher the order of Butterworth filter chosen the more ideal the gain curve will be. With that being said you cannot choose a higher-order filter easily as the accuracy of the filter decreases with an increase in the order. Hence it is best to choose the order of a filter while keeping an eye on the required accuracy.

Thank you